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Probing the gauge structure of high-temperature superconductors

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Abstract

We suggest that a spin-charge separating ansatz, leading to non-Abelian $SU(2)\otimes U_S(1)$ gauge symmetries in doped antiferromagnets, proposed earlier as a way of describing Kosterlitz-Thouless superconducting gaps at the nodes of the gap of d-wave (high-T_c) superconductors, may also lead to a pseudogap phase, characterised by the formation of (non-superconducting) pairing and the absence of phase coherence. The crucial assumption is again the presence of electrically charged Dirac fermionic excitations (holons) about the points of the (putative) fermi surface in the pertinent phase of the superconductor. We present arguments in support of the rôle of non-perturbative effects (instantons) on the onset of the pseudogap phase. As a means of probing such gauge interactions experimentally, we perform a study of the scaling of the thermal conductivity with an externally-applied magnetic field, in certain effective models involving gauge and/or four-fermion (contact) interactions.

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1 Introduction

Presently, there is an abundance of experimental data [1, 2, 3], indicating a very rich structure in the phase diagrams of the high-temperature superconducting cuprates. The pertinent phenomenology may be summarized by the temperature-doping-concentration phase diagram, shown in figure 1.

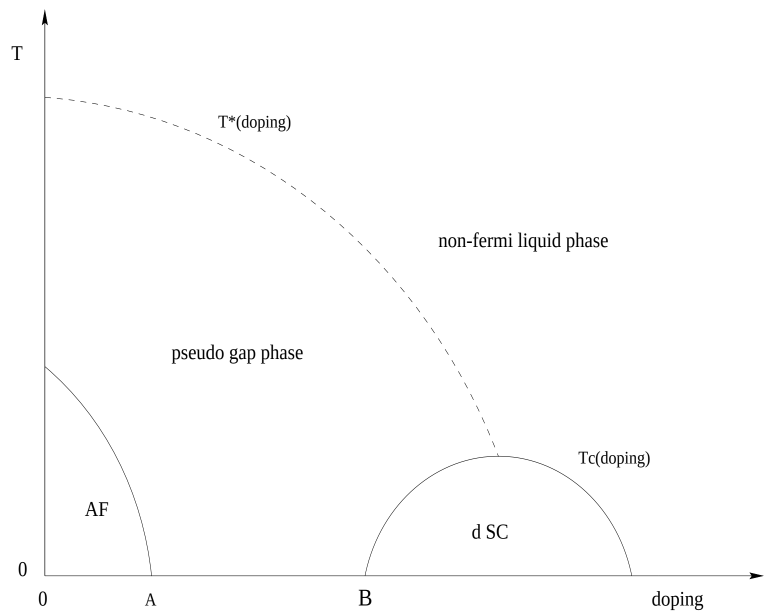

Figure 1: *The temperature-doping phase diagram summarizes the current (experimentally observed) situation in high-temperature superconducting cuprates. Notice the existence of an intermediate zero-temperature phase, characterised by the existence of preformed pairs, leading to a pseudogap.*

The phase diagram shows clearly a very-low (including zero) doping antiferromagnetic phase (AF). Above a critical doping concetration (point A in fig. 1), AF order is destroyed, but the interesting issue is the existence of a phase, named 'pseudogap phase', which interpolates between the AF and the superconducting phases (dSC), the latter being known to be of d-wave type [1].

It is a general belief today, supported by many experimental results [2] on optical conductivity, photo-emission, transport etc., that the pseudogap phase is characterized by pairing ('preformed pairs'), leading to the existence of a mass (pseudo)gap in the fermionic spectrum, which however is not accompanied by phase coherence. This situation is in sharp contradiction with the standard BCS theories of superconductivity, according to which phase coherence appears simultaneously with the gap.

The pseudogap phase is separated by a critical temperature curve T_c(doping) from the d-wave superconducting state, the latter being characterized by a sharp drop in resistivity, and strong type-II superconductivity [1], with the penetration depth for external magnetic fields of the order of a few thousands of Angströms. The pseudogap phase is also separated by another curve T^*(doping) from the non-fermi liquid normal state phase, where there is no gap, but there are abnormal properties, such as a linear dependence of the electrical resistivity for a wide range of temperature etc.

The physically challenging situation, therefore, for the high-temperature materials seems to be not so much their superconducting regime [1], but rather their behaviour in the normal state or under-doped regimes.

Several authors have made various proposals to explain the behaviour of the superconductor in those regimes. For our purposes here we shall take note of two approaches [4, 5], which are closer in spirit to the approach of refs. [6, 7], employing effective theories of relativistic fermions as the appropriate degrees of freedom for the dynamics of the underdoped (or pseudogap) phase of the cuprates.

In ref. [4], the adopted scenario is that the fermi surface of the underdoped cuprates consists of four small pockets, centered around $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ in momentum space. The analysis is made in the context of a spin-charge separating framework, and the charged excitations (holons) about the small pockets of the fermi surface are treated as relativistic charge excitations. In this latter respect the assumptions are apparently similar with those in our model [7]. However, the low-energy model used in that work, and the nature of the gauge symmetries involved in the spin-charge separation ansatz, are different from our model.

In the 'nodal liquid' approach of ref. [5], on the other hand, the starting

[1]Although, the main question why it is d-wave rather than s-wave superconductivity, has not, in our opinion, been answered yet.

point is the implementation of quantum disordering in the d-wave superconductor; the relevant excitations are relativistic Dirac fermions around the four points constituting the putative fermi surface in the underdoped situation [2], which however are electrically neutral, and hence, from our point of view, they correspond to spin degrees of freedom rather than holons. The lack of phase coherence in the pseudogap phase is then attributed in [5], by assumption, to a standard Kosterlitz-Thouless transition [8] of a planar superfluid.

In the apporach of ref. [7], relativistic electrically charged fermions are employed, as the pertinent degrees of freedom, describing the excitations about the nodes of the energy gap of a d-wave superconductor. The analysis pointed out the possibility of opening of a gap at the nodes of the d-wave superconducting gap, below a certain temperature, which however was much smaller ($T_c \leq 0.1$ K) than the critical temperature of the d-wave superconductor ($T_c^d \sim 100$ K). However, as pointed out in [9], upon the influence of an external magnetic field, the phenomenon of magnetic catalysis [10] was in operation [9], leading to a scaling of the dynamically induced 'nodal gap' with the magnetic field, and to an increase of the pertinent critical temperature up to $\sim 30^o$ K, for fields of order 10 Tesla [9] [3].

In this article, we point out that the gauge theory of ref. [7] can also provide a possible explanation of the pseudogap phase of fig. 1. We shall also point out that it is possible to distinguish experimentally the effects of gauge interactions from the ones due to four-fermi interactions among the 'nodal' holons, such as those considered earlier in ref. [11]. This can be done by measuring the thermal conductivity in the presence of external magnetic fields, following the experiments of [3]. We shall give details on the derivation of the scaling of the thermal conductivity with the externally applied magnetic field, based on the phenomenon of magnetic catalysis [10], in *both* the superconducting and pseudogap phases.

The structure of the article is as follows: in section 2 we describe the phase structure of the $SU(2) \otimes U_S(1)$ model, with emphasis on the rôle of instantons in inducing a pseudogap phase, without superconductivity and phase

[2]These points correspond to the d-wave gap nodes in the superconducting phase.

[3]Note that the high-temperature superconductors are known to be strongly type II, with a London-Meissner penetration depth of a few thousands of Angströms, thereby justifying the analysis in the presence of an external magnetic field, even in the superconducting state.

coherence. In section 3 we discuss the behaviour of the relativistic charged fermion excitations, argued to describe the pertinent excitations about the nodes of the (putative) fermi surface in the pseudogap phase, in the presence of external magnetic fields. Then, we study the scaling of the thermal conductivity with the magnetic field, for various interactions (gauge and four-fermion type) among the fermions that may characterise the model in various regimes of the phase diagram. If our model is correct, and electrically charged relativistic excitations do indeed play a rôle in the pseudogap phase, then, such a difference in scaling should be seen in experiments like those of ref. [3], when applied to the pseudogap phase of the cuprates. Finally, conclusions and outlook are presented in section 4, where we also discuss other ways of detecting the presence of gauge interactions in the high-T_c materials.

2 Non-perturbative Effects in the $SU(2) \otimes U_S(1)$ Gauge Theory and the Pseudogap phase

The important feature of the spin-charge separating ansatz of ref. [7] was its non-Abelian hidden local gauge symmetries, $SU(2) \otimes U_S(1)$, emanating from a 'particle-hole' representation of the electronic degrees of freedom [12], even in the case of finite doping concentrations. The mass gap of the model, and hence the pairing between the charged excitations (holons), occurs due to the statistics changing $U_S(1)$ interactions, which is an exclusive feature of the planar geometry of the cuprate materials. The presence of such interactions in the spin-charge separation ansatz may be understood [7] by means of bosonization techniques in three dimensions [13]. The mass generation breaks the $SU(2)$ group down to a $U(1)$ subgroup. We now note that the $U_S(1)$-induced mass gap is characterised by the absence of a local order parameter. This occurs even in the zero-temperature $(2+1)$-dimensional field theory, and it is a characteristic feature of the Kosterlitz-Thouless (KT) nature [8] of gauge theories in $2+1$ dimensions, as argued in [14, 6].

When applied to the non-Abelian gauge model of [7], the above symmetry-breaking mechanism leads to unconventional KT superconductivity, provided that the gauge boson of the unbroken $U(1) \in SU(2)$ is *massless*. Due to the compactness of the $U(1)$ gauge group, however, which is a distinctive feature of the non-Abelian gauge group of the model [7], there are non-perturbative

effects (instantons), which are responsible for giving the gauge boson of $U(1)$ a small but finite mass [15, 16]. This spoils superconductivity, leaving only a phase, characterised by pairing among the holons, without the existence of phase coherence. It is one of the points of this article to argue that such a phase may provide a possible explanation about the 'pseudogap ' phase of high-temperature superconductivity [2].

Let us see this in more detail. The effective low-energy (continuum) theory, describing charged excitations about the nodes of a d-wave superconducting gap [7], or the points of the putative fermi surface in the underdoped situation, after the integration of magnon (spin) degrees of freedom, reads:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(\mathcal{G}_{\mu\nu})^2 + \overline{\Psi}D_\mu\gamma_\mu\Psi + \kappa\sum_{a=1}^{2}\left(\overline{\Psi}_a\Psi^a\right)^2 \tag{1}$$

where Ψ_a are the relativistic spinors describing the excitations about the nodes of a d-wave gap, $D_\mu = \partial_\mu - ig_1 a_\mu^S - ig_2\sigma^a B_{a,\mu}$, and $F_{\mu\nu}$, $\mathcal{G}_{\mu\nu}$ represent the field strengths for the $U_S(1)$, $SU(2)$ gauge groups respectively. For simplicity in this work we work in units $\hbar v_F = 1$, with v_F the fermi velocity for holons, which plays the rôle of the velocity of light in the relativistic formalism [4]. The four-fermion interactions are assumed attractive, with $\kappa > 0$.

First, let us ignore the four-fermion interactions, assuming them irrelevant. The gauge $U_S(1)$ interaction is capable of inducing dynamical opening of a holon gap (pairing) if the pertinent coupling constant of the statistical model lies inside the $SU(2)$ broken regime of the phase diagram of fig. 2, which is derived in [17].

An important feature for the onset of a pseudogap is that the non-Abelian gauge group $SU(2)$ breaks down to a compact $U(1)$, generated by the σ^3 Pauli generator of the $SU(2)$ group, as we discussed previously [7, 17]. Due to the electric charge of the fermions Ψ, the coupling with $U_E(1)$ of electromagnetism causes a spontaneous breaking of the electromagnetic symmetry, as can be seen by considering the following matrix element:

$$\mathcal{S}^a =< B_\mu^a|J_\nu|0>,\ a = 1,2,3\ ; \qquad J_\mu = \overline{\Psi}\gamma_\mu\Psi \tag{2}$$

It should be stressed that as a result of the colour group structure only the massless B_μ^3 gauge boson of the $SU(2)$ group, corresponding to the σ_3

[4]In realistic situations [9], this velocity is smaller than the velocity of light c entering the electromagnetic interactions. However, for the qualitative purposes of this work we shall absorb c in the definition of the electric charge e.

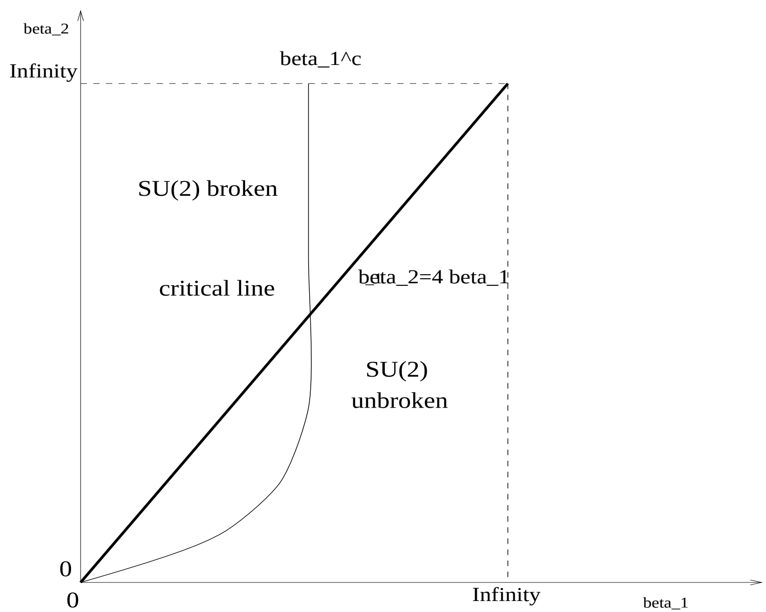

Figure 2: *Phase diagram for the $SU(2) \times U_S(1)$ gauge theory. The thick straight line indicates the specific relation of the coupling constants in the statistical model for doped antiferromagnets used in the present work.*

generator in two-component notation, contributes to the graph. The result is [14, 6, 7]:

$$\mathcal{S} =< B^3_\mu | J_\nu | 0 >= (\mathrm{sgn} M) \epsilon_{\mu\nu\rho} \frac{p_\rho}{\sqrt{p_0}} \tag{3}$$

where M is the parity-conserving fermion mass (or the holon condensate in the context of the doped antiferromagnet). As discussed in [6, 14] the B^3_μ colour component plays the rôle of the Goldstone boson of the (global) $U_E(1)$ symmetry [6] [5].

If the gauge boson B^3_μ of the unbroken $U(1)$ subgroup of $SU(2)$ remained *massless* exactly, then it would be responsible for the appearance of a massless pole in the electric current-current correlator [6], which would be the characteristic feature of any superconducting theory. The question is

[5] After coupling with external electromagnetic potentials, of course, the (global) fermion-number symmetry $U_E(1)$ is gauged and one has the Anderson-Higgs phenomenon of symmetry breaking, as explained in [6].

whether such a pattern of symmetry breaking is capable of explaining the non-superconducting intermediate (zero-temperature) phase AB of fig. 1. At first instance, this seems not possible, in view of the appearance of a massless pole in the electric current-current correlator [6].

However, in the model of [7], the compact abelian subgroup $U(1) \in SU(2)$, may contain *instantons* [16], which in three space-time dimensions are like monopoles, and are known to be responsible for giving a *small* but *non-zero mass* to the gauge boson B_μ^3,

$$m_{B^3} \sim e^{-\frac{1}{2}S_0} \tag{4}$$

where S_0 is the one-instanton action, in the dilute gas approximation. Such a small mass is sufficient to destroy superconductivity of the model. This however, from the point of view discussed in this section, is a welcome result, given that it explains naturally the existence of 'pre-formed pairs', and the opening of a gap (amplitude of the putative order parameter), but does not lead to superconductivity, in agreement with the phenomenology of the pseudogap phase.

We now come to the four-fermion interactions. The attractive four-fermion interactions ($\kappa > 0$) in (1) have been taken for simplicity to be of the Gross-Neveu type. They arise naturally in the statistical models of interest to us here, e.g. as describing the tendency of holes to lie on neighbouring sites of the antiferromagnetic lattice [18, 6] or describing other contact interactions, appropriate for effective theories of the Hubbard model [11]. From the pure field-theoretic view point, it is worth mentioning that such four-fermion interactions in $(2+1)$-dimensions become renormalizable (relevant operators) in the context of a $1/N$ framework, where N is a flavor number for fermions [19].

In the presence of $U_S(1)$ and four-fermi interactions (ignoring the $SU(2)$ interactions for simplicity), the pertinent phase diagram of the model (1) has been derived, within a $1/N$ framework, in ref. [6], and is depicted in figure 3.

We now remark that in the context of the statistical model of ref. [7], which will be the basis of our discussion here, the various couplings depend explicitly on the doping concentration in the sample. In view of the phase diagrams of figs. 2, 3, this implies that by tuning the doping concentration appropriately, one can induce phase transitions in the model.

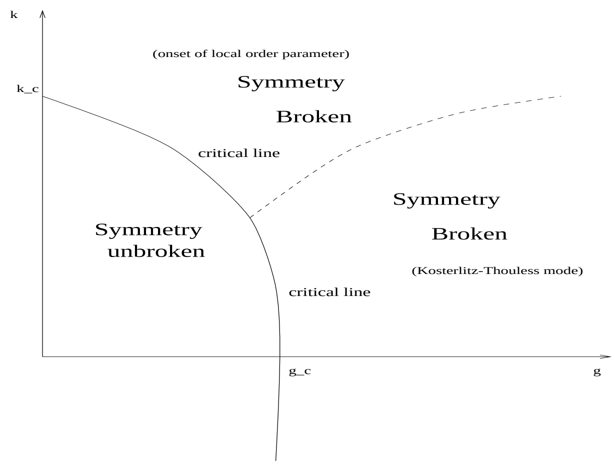

Figure 3: *A generic phase diagram of the theory with* $U_S(1)$ *gauge (g) and four-fermion (*κ*) (Gross-Neveu) interactions. The critical line separates the phase of unbroken symmetry from that of broken symmetry. The symmetry breaking is due to the fermion condensate. The dashed line indicates the (possible) onset of a long-range order (local order parameter) due to the dominance of four-fermion interactions. The shape of the line is conjectural (at present).*

For instance, the inverse couplings of the $SU(2)$ and $U_S(1)$ gauge groups lie in the thick straight line depicted in fig. 2. At present, the precise shape of the critical line is not known [17], since it requires the construction of an appropriate fermionic algorithm, which will allow for a proper lattice study of the model. However, the strong coupling analysis of [17] has demonstrated that the critical line passes through the origin of the graph. For our qualitative purposes, it will be sufficient, and most likely physically correct, to assume an almost vertical shape of the critical line from near the upper intersection point with the thick straight line (away from the origin) till its intersection with the ($\beta_2 = \infty$)-axis. This implies that the critical coupling for mass generation for the coupling β_1 in the statistical model is still given

by the $U_S(1)$ gauge theory critical coupling, i.e. $\beta_1 < \beta_1^c$ (see figure 1). It is known that $(\beta_1^c)^{-1} \sim \pi^2/32$ [20, 21]. Hence, taking into account the fact that $\beta_1^{-1} \sim \frac{J}{\Lambda}(1-\delta)$ in the model of [7], where δ is the doping concentration in the sample, and Λ is an appropriate ultraviolet cut-off, one obtains that pairing due to gauge interactions occurs for:

$$\delta_{AF} < \delta < \delta_c^{(2)} \equiv 1 - \frac{\pi^2}{32}\frac{\Lambda}{J} \tag{5}$$

where δ_{AF} denotes the doping concentration at which the AF order is destroyed [6]. In view of the previous discussion, this region corresponds to the pseudogap phase of the diagram of fig. 1.

The onset of (d-wave) superconductivity may occur at higher doping concentrations, for which the attractive four-fermi couplings among charged excitations in the effective field theory become strong enough, so as to overcome the gauge interactions, and lead to a standard BCS-type pairing among the charged excitations.

From standard arguments [19], we know that pair formation, and hence mass generation, in four-fermion Gross-Neveu theories occurs for dimensionless inverse couplings, $\lambda \equiv \frac{1}{2\kappa\Lambda}$ weaker than a critical value [7], $2/\pi^2$. However, the full phase diagram, incorporating the $SU(2)$ and $U_S(1)$ couplings as well, as appropriate for the model of [7], will be more complicated. However, for our purposes in the present work it will be sufficient to consider only the effects of the $U_S(1)$ coupling, responsible for the mass generation in the model.

In toy models of doped antiferromagnets [24, 11], the coupling constant κ depends on δ:

$$\kappa \propto \kappa_0(t', J')\frac{1}{1-\delta} \tag{6}$$

where $\kappa_0(t', J')$ is an appropriate function of the next-to-nearest-neighbor hopping element, and Heisenberg exchange energies.

[6] The existence of δ_{AF} in space-time dimensionality higher than two, may de inferred by a renormalization-group analysis [22] of the phase diagram of the magnon (CP^1) sector of the model of [7]. For our purposes here we shall not deal with this sector explicitly.

[7] This, of course, pertains to the model in the absence of external magnetic fields. The magnetic catalysis phenomenon [10] changes the situation [9, 23], as it may lead to four-fermion-interaction induced pairing even for subcritical (weak) contact interactions. We shall deal with these issues in the next section.

When combined with the phase diagram of fig. 3, this implies that pairing due to four-fermion interactions would occur for doping concentrations in a region determined by the critical line of fig. 3. For instance, in the simplifying case when the gauge coupling is weak enough, so that mass generation is due to the four-fermi interactions only, this would give:

$$\delta > \delta_c^{(3)} \equiv 1 - \frac{4\Lambda\kappa_0(t', J')}{\pi^2} \tag{7}$$

By appropriately choosing $\kappa_0(t', J')$, it is possible to arrange for a situation like the one depicted in fig. 1, where the zero-temperature pseudogap phase interpolates between the AF and the standard BCS-type d-wave superconducting theory.

Notice that the dynamical mass generation due to four-fermi couplings leads to a *second order* phase transition, at zero temperature, and hence to phase coherence (local order parameter), as is standard in BCS-like pairing. This should be contrasted with the situation in the case of gauge interactions, described above, which leads to Kosterlitz-Thouless type of breaking [14, 6, 7] even at zero temperature [8].

Before closing this section we cannot resist in pointing out that there exist some alternative possibilities for the gauge interactions, which may lead to interesting phase structure in the superconducting region of the diagram of fig. 2. In one possible scenario, as one increases the doping concentration from the pseudogap phase, a point is reached in the δ axis, where there is a special relation among the various coupling constants of the effective spin-charge separating theory, leading to a $N = 1$ supersymmetry [25] [9]. For example, in the context of models of ref. [24], such a supersymmetric point could be reached for doping concentrations $t' \sim \sqrt{JJ'}(1-\delta)^{3/2}$. Supersymmetry is known [16] to suppress instanton contributions, in the sense that the instanton-induced mass of the B_μ^3 gauge boson is now given by:

$$m_{B^3} \sim e^{-S_0} \tag{8}$$

[8] The finite temperature situation is not discussed here, however one should mention that the coupling of superconducting planes may be necessary in maintaining a phase coherence for the four-fermi theory at finite temperatures.

[9] This supersymmetry carries non-trivial dynamical information about the spin-charge separation mechanism underlying the model, and hence it is different from the non-dynamical global supersymmetry algebras, at specific points of the coupling constants, discovered in [26].

At such a point, the suppression may be sufficient to allow for a gauge-theory induced Kosterlitz-Thouless (KT) superconducting gap at the d-wave nodes. The KT nature of the gap implies that once opened such a gap cannot affect the d-wave character. This scenario for superconductivity has been advocated in ref. [7].

Once a supersymmetric point is reached, there is an alternative scenario [25], which could be in operation in the superconducting regime of fig. 2 for our model. In supersymmetric theories of the type considered here and in ref. [25], it is known [16] that supersymmetry cannot be broken, due to the fact that the Witten index $(-1)^F$, where F is the fermion number, is always non-zero. Thus, in supersymmetric theories the presence of instantons should give a small mass, if at all, in both the gauge boson and the associated gaugino, the latter having been argued in [25] to represent the coupling between the superconducting planes.

However, in three-dimensional supersymmetric gauge theories it is possible that supersymmetry is broken by having the system in a 'false' vacuum [16], where the gauge boson remains massless, even in the presence of non-perturbative configurations, while the gaugino acquires a small mass, through non-perturbative effects. The lifetime, however, of this false vacuum is very long [16], and hence superconductivity can occur, in the sense that the system will remain in the false vacuum for a very long period of time, longer than any other time scale in the problem. A massless gauge boson would imply superconductivity as we discussed above, in the sense of the appearance of a massless pole in current-current correlators. In such a scenario, the false vacuum would occur in the dSC region of the phase diagram, at the nodes of the d-wave gap. The KT nature of the superconductivity would imply that the d-wave character of the fermi surface of the complete statistical system is not affected by the opening of a gap at the nodes.

We now point out that, experimentally, one can make a distinction between a gap induced by the gauge interactions, or by four-fermi interactions, as a result of the different scaling of the mass gap with an externally applied magnetic field. A suggestive experiment along these lines is that of ref. [3], measuring the behaviour of the thermal conductivity, in both the superconducting and 'pseudogap ' phases. Theoretically, one needs to develop a formalism for the study of the dynamical opening of a gap (fermion condensate) in the presence of an external magnetic field, a phenomenon known as magnetic catalysis [10]. This will be the topic of the subsequent sections.

3 Interacting Fermions in external magnetic fields

3.1 Review of the Basic Formalism

In this section we review briefly the theoretical formalism underlying the behaviour of $(2+1)$-dimensional relativistic fermion systems under the influence of external magnetic fields [10, 27, 28]. One follows essentially the method of Schwinger [29], by looking at the coincidence limit of the fermion propagator (in configuration space), $\mathrm{Lim}_{x\to y}\ S(x,y)|_B$, in the presence of a constant external field, B. We start first from the free-fermion case, i.e. the case where the fermions interact only with the external constant field. The external gauge potential is given by:

$$A_\mu^{ext} = -Bx_2\delta_{\mu 1} \tag{9}$$

and the Lagrangian is:

$$L = \frac{1}{2}\overline{\Psi}(i\gamma^\mu(\partial_\mu - ieA_\mu^{ext}) - m)\Psi \tag{10}$$

where m is a *parity conserving* fermion mass, and the γ-matrices belong to the reducible 4×4 representation, appropriate for an even number of fermion species [20, 6, 7].

For our phenomenological purposes in this work, we shall assume that the mass m is generated *dynamically* by *either* the $U_S(1)$ strong interactions [7] *or* the four-fermion contact interactions [10, 23], in (1). The phenomenon of magnetic catalysis [10] implies that there will be a scaling of the fermion condensate with the externally applied field, in such a way that dynamical mass generation may occur for arbitrarily weak couplings, when the external field is sufficiently strong.

Following the proper time formalism of Schwinger [29], the fermion propagator $S(x,y)|_B =< 0|T\overline{\Psi}(x)\Psi(y)|0> |_B$ in the presence of a constant external magnetic field, B, can be calculated exactly [10]. The expansion of the (Fourier transform of the) Euclidean propagator in terms of the Landau level contributions is given by [30, 10]:

$$\tilde{S}_E(k)|_B = -ie^{\frac{-k_\perp^2}{eB}}\sum_{n=0}^{\infty}(-1)^n\frac{D_n(m,B,k)}{k_3^2+M_n^2(B)}, \tag{11}$$

with $M_n^2(B) \equiv m^2 + 2eBn$, and

$$D_n(eB,k) = (m - k_3\gamma_3)[(1 - i\gamma_1\gamma_2 \mathrm{sgn}(eB))L_n^0(2\frac{k_\perp^2}{|eB|})$$
$$-(1 + i\gamma_1\gamma_2 \mathrm{sgn}(eB))L_{n-1}^0(2\frac{k_\perp^2}{|eB|})]$$
$$+4(k_1\gamma_1 + k_2\gamma_2)L_{n-1}^1(2\frac{k_\perp^2}{|eB|}) \tag{12}$$

with $L_n \equiv L_n^0$, $L_{-1}^1 = 0$, $L_{-1} = 0$. The normalization of the Laguerre polynomials is taken as follows [10, 31]: $\int_0^\infty e^{-x} L_n(x) L_n(x) dx = 1$.

For our purposes in this work it is useful to quote the result of the (regularized) magnetically-induced fermion condensate at zero temperature, after summation of the Landau levels [28], in the case of small bare fermion mass $m << \sqrt{eB}$:

$$\Delta < \overline{\Psi}\Psi > |_{T=0} = \frac{eB}{2\pi}\left(1 + \zeta(\frac{1}{2})\frac{\sqrt{2}m}{\sqrt{eB}} + \mathcal{O}(m^3/(eB)^{\frac{3}{2}})\right) \tag{13}$$

The finite temperature analysis has been given in [28], where we refer the interested reader for details. The summation over the Landau poles is possible to be carried out analytically in some cases. For instance, for a strong magnetic field, B, and low temperatures such that $eB >> T^2 >> m^2$, one has: $\Delta < \overline{\Psi}\Psi > |_T \simeq \frac{eB}{4\pi}\frac{m}{T} - \frac{1.46m\sqrt{eB}}{\sqrt{2}\pi}$, using $\zeta(1/2) = -1.46$. This implies the existence of a *critical temperature*, for non-zero m,

$$T_c \simeq \frac{1}{4}\sqrt{eB} \tag{14}$$

above which the condensate vanishes. The order of magnitude of the temperature is consistent with the approximations made in the derivation of (14), which suggests an important rôle for the higher Landau levels at finite temperatures in inducing a finite critical temperature even for the free-fermion case. In the context of a possible application of this phenomenon to high-temperature superconductors [9], we note that $\sqrt{eB}$ scaling of a critical temperature with the external field intensity are reported in the experiments of [3].

Notice however, that in the presence of extra interactions that may lead to dynamical mass generation for fermions, as happens in the $SU(2) \times U_S(1)$ model of [7], the scaling of the critical temperature with B may be different [9]. This will be crucial for probing such sturctures experimentally, as we discuss in the next subsection.

Consider first the magnetically catalysed condensate in the case where there is dynamical mass generation by means of $U_S(1)$ gauge interactions [9]. For strong magnetic fields $\sqrt{eB} >> \alpha$, the lowest-Landau level truncation in the Schwinger-Dyson analysis proves sufficient. The pertinent low-temperature gap equation reads [9, 32]:

$$1 = \alpha \int_0^\infty dx e^{-l^2 x/2} \frac{1}{[(\pi T)^2 + x - m^2(T)]^2 + (2\pi T)^2 m^2(T)}$$
$$[\frac{(\pi T)^2 + x - m^2(T)}{m(T)} \tanh \frac{m(T)}{2T} + \frac{(\pi T)^2 + m^2 - x}{\sqrt{x}} \coth \frac{\sqrt{x}}{2T}] \quad (15)$$

where α is the (dimensionful) fine-structure constant of the $U_S(1)$ theory, T is the temperature, and l is proportional to the magnetic length, $l^2 \equiv 1/eB$. By following the same semi-analytic procedure as in ref. [9], it is easy to see that for strong enough fields, and low enough temperatures we are interested in here, $T << m$, the gap equation simplifies to:

$$m/\alpha \simeq (\tanh(m/2T) - (2T/m))\, e^{-\frac{m^2}{2eB}} \int_{-1}^0 \frac{dz}{z} e^{-\frac{m^2}{2eB} z} +$$
$$[(\tanh(m/2T) - (2T/m))\, e^{-\frac{m^2}{2eB}} + 2T/m] \int_0^\infty \frac{dz}{z} e^{-z} \quad (16)$$

Regularizing the infinities of the integrals around $z \sim 0$, as in ref. [9], in the limit $T/m \to 0$, one obtains:

$$m \sim \alpha[\tanh \frac{m}{2T} - \frac{2T}{m}] e^{-\frac{m^2}{2eB}} \ln\left(\frac{2eB}{m^2}\right) \quad (17)$$

which for strong enough magnetic fields and low temperatures, leads to the following (approximate) expression for the (temperature dependent) mass gap:

$$m_b^g(B,T) = m_B(B,0) - |\mathcal{O}(T/m)| = C\alpha \ln \frac{\sqrt{2eB}}{\alpha} - |\mathcal{O}(T/m)| \quad (18)$$

where $m_B(B, 0)$ is the zero-temperature solution [9, 27]. A numerical estimate of $C \sim \sqrt{2}$ has been made in ref. [9] by a zero-temperature analysis. The solution (18) is consistent with the the existence of a critical temperature T_c, which has been estimated in [9] to be:

$$T_c \propto \alpha \ln \frac{\sqrt{2eB}}{\alpha} \tag{19}$$

The formula (18) suggests that for the low-temperature regime $T << T_c$, and strong magnetic fields, the replacement $m_g(T, B) \sim m(0, B)$ proves sufficient for an estimate of the leading scaling behaviour of the thermal conductivity with B.

We next remark that in the case of the subcritical Gross-Neveu model [10, 23] the corresponding zero-temperature mass gap for strong magnetic fields scales as:

$$m_b^{NJL} \sim 0.45\sqrt{eB} + \ldots \tag{20}$$

where the $\ldots$ indicate subleading terms, proportional to the deviation of the four-fermion coupling from its critical value [19]. At present the analysis for the supercritical four-fermion model is not available, but it presents no conceptual issues. As in the gauge case, the gap (20) will be used for an estimate of the low-temperature thermal conductivity scaling.

As we shall show below, in this region of the parameters, the scaling of the thermal conductivity in the case of the $U_S(1)$-induced gap [7] is different from that of the free-fermion [28] and four-fermi Gross-Neveu models [10, 23].

3.2 Thermal Conductivity

The expression for the thermal conductivity can be given in terms of the energy-current (momentum) correlation function [33]:

$$\kappa^{el}(\omega) = \frac{\beta}{V}\int_0^\infty dt \int_0^\beta d\lambda \mathrm{Tr}\{\rho_0 P^i(0) P^j(t + i\lambda)\} e^{-i\omega t} \tag{21}$$

where V is the volume of the system, $\beta = 1/T$ is the inverse temperature (in units $k_B = 1$), and $P(t) = e^{iHt}P(0)e^{-iHt}$, $P^i(0) = \frac{i}{2}\int d^2x\ (\overline{\Psi}\gamma^0\partial^i\Psi - \partial^i\overline{\Psi}\gamma^0\Psi)$, with H the Hamiltonian; $\rho_0 = \frac{1}{Z}\mathrm{Tr}e^{-\beta H}$ is the Gibbs equilibrium distribution, Z is the partition function of the system, and the trace Tr is taken over all physical states.

Following ref. [23], one can compute the static thermal conductivity of an isotropic system, which will be of interest to us here:

$$\kappa^{el} = \frac{1}{4T}\lim_{\omega\to 0}\frac{1}{\omega}\left(G(p=0, i\nu_m = \omega + i0^+) - G(p=0, i\nu_m = \omega - i0^+)\right). \tag{22}$$

where the thermal Green function can be calculated perturbatively in the theory [23]:

$$G(p=0, i\nu_m) = iT\sum_{n=-\infty}^{+\infty}\int \frac{d^2k}{(2\pi)^2}k^2 \mathrm{tr}\left(\gamma^0 S(i\omega_n, k)\gamma^0 S(i\omega_n + i\nu_m, k)\right), \tag{23}$$

We follow ref. [23] and use the following spectral representation for the fermion thermal Green function: $S(i\omega_n, k) = \int\limits_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{a(\omega,k)}{i\omega_n - \omega}$, where $a(\omega, k) = 2ImS(i\omega_n = \omega - i0^+, k)$. Summation over the Landau levels implies:

$$\begin{aligned}
&a(\omega, k) = 2\pi\mathrm{sgn}(\omega)\sum_{n=0}^{\infty}(-1)^n\delta(\omega^2 - M_n^2(eB))(m + \omega\gamma_0)e^{-\frac{k_\perp^2}{eB}} \otimes \\
&[(1 - i\gamma_1\gamma_2\mathrm{sgn}(eB))L_n(2\frac{k_\perp^2}{|eB|}) - \\
&(1 + i\gamma_1\gamma_2\mathrm{sgn}(eB))L_{n-1}(2\frac{k_\perp^2}{|eB|})] + \\
&8\pi\mathrm{sgn}(\omega)\sum_{n=0}^{\infty}(-1)^n\delta(\omega^2 - M_n(eB)^2)e^{-\frac{k_\perp^2}{eB}}(-i\gamma_1 k_1)L_{n-1}^1(2\frac{k_\perp^2}{|eB|}) + \\
&4\sum_{n=0}^{\infty}(-1)^n P(\frac{1}{\omega^2 - M_n^2(eB)})e^{-\frac{k_\perp^2}{eB}}(-k_2\gamma_2)L_{n-1}^1(2\frac{k_\perp^2}{|eB|})
\end{aligned} \tag{24}$$

Substituting this spectral representation into the spectral representation for $S(i\omega_n, k)$, using the result into Eq. (23), and performing the sum over the Matsubara frequencies, we arrive at the following expression for the thermal conductivity [23]:

$$\kappa^{el} = \frac{1}{16T^2}\int\frac{d^2k}{(2\pi)^2}\int\frac{d\omega}{2\pi}\frac{k^2}{\cosh^2\left(\frac{\omega}{2T}\right)}\mathrm{tr}\left(\gamma^0 a(\omega,k)\gamma^0 a(\omega,k)\right). \tag{25}$$

Substituting (24) in (25), we obtain after some tedious algebra:

$$\kappa^{el} = \frac{3}{16}\frac{(eB)^2}{T^2}\delta(0)\sum_{n=1}^{\infty}\frac{n}{\cosh^2(\frac{M_n}{2T})} + \frac{(eB)^2}{16T^2}\delta(0)\frac{1}{\cosh^2(\frac{m}{2T})} \tag{26}$$

Above we have used the following mathematical identities for the δ-functions:

$$\delta\left(\omega^2 - M_1^2\right)\delta\left(\omega^2 - M_2^2\right) = \frac{1}{4M_1M_2}\left(\delta\left(\omega + M_1\right)\delta\left(\omega + M_2\right) + \delta\left(\omega - M_1\right)\delta\left(\omega - M_2\right)\right) \qquad (27)$$

by means of which the double summation over Landau levels is reduced to a single one. We have also made use of the identities of the Laguerre polynomials [31]:

$$\int_0^\infty e^{-x} x L_n(x) L_n(x) dx = 2n+1, \quad \int_0^\infty e^{-x} x^2 L_n^1(x) L_n^1(x) dx = 2(n+1)^2 \; . \qquad (28)$$

Now we are in a position to state the main result of the present work, namely to estimate the scaling of κ^{el} with the magnetic field, for the two cases of dynamical mass generation, induced by gauge [7, 27] or four-fermion fields [10, 23], the latter interaction being taken to be of the Gross-Neveu type for simplicity. Following [23], we also replace $\delta(0)$ in the expression for the thermal conductivity (26) by the inverse of the width Γ for the quasiparticle states, $1/\Gamma$, where Γ is assumed considerably smaller than the dynamical mass, $\Gamma << m$, in order to preserve the validity of the symmetry breaking analysis due to the magnetic catalysis.

The pertinent results are summarised in figs. 4, 5 and 6, for the cases of (a) free relativistic fermions in an external field, with vanishing bare mass as compared to $\sqrt{eB}$ [28], (b) sub-critical Gross-Neveu model [23], and (c) $U_S(1)$ interactions [7, 6], respectively. The figures show the thermal conductivity versus the applied magnetic field, and - in order to facilitate the comparison - we have expressed the various scales in units of the dimensionful structure constant α of the $U_S(1)$ interactions in all three cases.

The analysis in all cases has been made at a fixed low temperature, which allows some analytic results to be derived from the respective formalism of refs. [10, 28, 9, 23]. From the results of the previous subsection, it is clear that the critical temperatures T_c (14), (19) for the magnetic catalysis phenomenon [9, 28] scales with the magnetic field appropriately in each case. Thus, for fixed $T < T_c$, as is the case in the figures, this implies a minimum B above which the magnetic catalysis phenomenon occurs. This has been taken properly into account in drawing the figures.

In the free fermion case, summation over *all Landau levels* in the expression for the thermal conductivity is possible [28]. In the case of vanishing

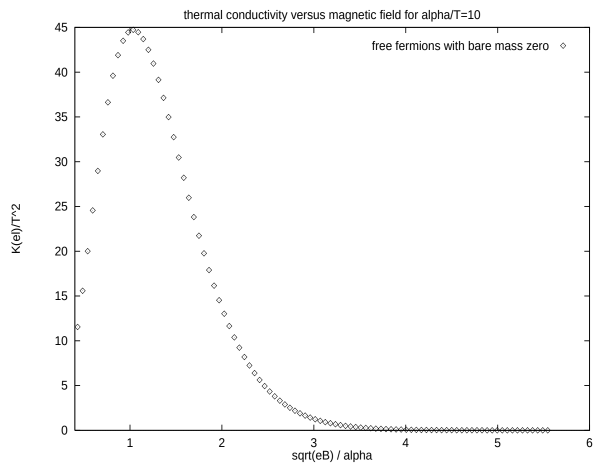

Figure 4: *Scaling of the thermal conductivity for relativistic electrically charged fermions with a magnetic field, in the free case, with vanishing bare mass of the fermions, as compared to the* $\sqrt{eB}$. *The temperature and mass scales are all given in units of the* $U_S(1)$ *gauge structure constant, to make easier the comparison with the gauge case.*

bare mass, as compared to $\sqrt{eB}$, the resulting magnetically-induced condensate at any finite temperature is given by the zero-temperature result (13) [28], provided that $T < T_c$, with T_c given by (14). The thermal conductivity shows a peak. If one takes the behaviour of fig. 4 literally, then, the presence of a low-magnetic field region, where the conductivity increases with the magnetic field, contradicts the experimental situation of ref. [3] in the superconducting phase of the high-T_c cuprates, where the conductivity reduces with the magnetic field for low fields. This seems to point towards the fact that other interactions set in in such regimes.

In the subcritical four-fermion (Gross-Neveu) case [23], the scaling for strong magnetic fields, is similar to the free fermion case.

In the gauge case, on the other hand, the logarithmic scaling of the mass gap (18) with the external field is a distinctive feature, and leads to a different scaling for the thermal conductivity, as compared to the previous cases. This is clear from the figs. 5,6. We also note that for the regime of the tem-

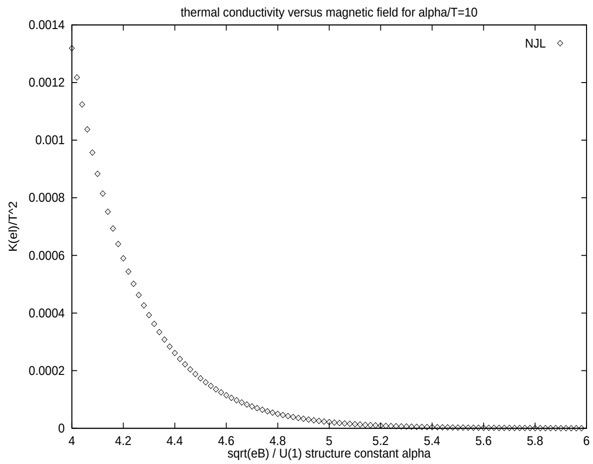

Figure 5: *Scaling of the thermal conductivity for relativistic electrically charged fermions with a magnetic field, in the case of sub-critical Gross-Neveu four-fermion interactions. The diagram pertains to strong magnetic fields and low-temperatures, so that the lowest-Landau level analysis is reliable. The temperature and mass scales are all given in units of the* $U_S(1)$ *gauge structure constant.*

peratures considered in fig. 6, the condition $T < T_c$, or equivalent B larger than a minimum value, implies that the peak which occurs in the thermal conductivity versus B (as in the case of fig. 4), even in the gauge case, lies outside the allowed region of B.

The interesting question is whether the gauge interactions lead to a scaling of the thermal conductivity in the low-field region which agrees with the results of ref. [3]. Unfortunately, at present, analytical results are known only for the case of very strong fields, where only the lowest Landau level contribution is taken into account. Further studies along this direction are in progress.

However, even at this preliminary stage, our analysis above has demonstrated that in principle, thermal conductivity experiments, under the influence of external magnetic fields, may be useful probes of gauge structures in the dynamics of high-temperature superconductors, provided one performs

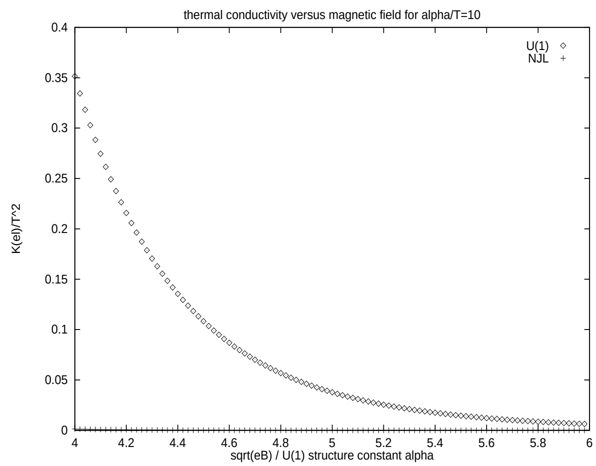

Figure 6: *Scaling of the thermal conductivity for relativistic electrically charged fermions with a magnetic field, in the $U_S(1)$-gauge-interaction case. The diagram pertains to strong magnetic fields and low-temperatures, so that the lowest-Landau level analysis is reliable. For comparison we have also included the result for the Gross-Neveu model (see previous figure). The temperature and mass scales are all given in units of the $U_S(1)$ gauge structure constant.*

the experiments in different regions of the phase diagram of fig. 1. If there is a regime of doping for which gauge interactions set in, then there should be a difference in scaling of the thermal conductivity with the external field, as compared to the regime where the standard BCS type four-fermion interactions among the quasiparticles are present.

It is of course, understood, that our preliminary analysis above pertains only to the thermal conductivity of relativistic, electrically charged, 'nodal' quasi-particle excitations, which have been argued to play a rôle in *both* the pseudogap as well as the d-wave superconducting phases [9]. In actual experiments, the total thermal conductivity receives contributions from phonons etc, and this should be taken properly into account in all the expressions above. However, this falls beyond our scope here.

4 Conclusions

In this article we have presented arguments supporting the rôle of non-perturbative effects (instantons) in the non-Abelian $SU(2) \otimes U_S(1)$ model for high-temperature superconductors, discussed in [7], on the appearance of a 'pseudogap ' phase, lying between the antiferromagnetic and the d-wave superconducting phases (c.f. fig. 1). The instantons have been argued to give a small mass to the gauge boson of the $U(1) \in SU(2)$, which otherwise would have played the rôle of the massless pole of the superconducting phase. The Kosterlitz-Thouless nature of the symmetry breaking [6, 7] was held responsible for the absence of phase coherence, which characterises the pseudogap phase.

The basic feature of the model is the relativistic nature of the charged excitations, relevant for the description of quasiparticles about the four points constituting the (putative) fermi surface in the pseudogap phase [9, 5].

We have also argued that above a given doping concentration, higher than the ones at which gauge interactions set in, four-fermi contact interactions become relevant, and they induce a gap in the quasiparticle spectrum. For simplicity we have considered Gross-Neveu type interactions, although the analysis can be extended to other types.

We have pointed out that, experimentally, such interactions may be distinguished from the gauge interaction by observing a change in the scaling properties of the thermal conductivity of the nodal quasiparticles with an externally applied magnetic field, for fixed temperatures, as the doping concentration changes. We have suggested the extension of the experiments of ref. [3] in the region of the pseudogap phase as well, because it is in this region that the gauge interactions have been argued to be relevant for the opening of a Kosterlitz-Thouless gap for the nodal excitations. We should note at this point that if the magnetic catalysis phenomenon occurs in the pseudogap phase, in much the same way as it occurs in the superconducting case of ref. [3], then, according to the discussion in this article, there is a significant possibility that the dynamics of this phase is dominated by *electrically charged* nodal excitations, similar to the ones about the nodes of the d-wave superconducting gap suggested in [9], contrary to current theoretical scenaria [5].

Before closing we would like to remark that an alternative way of probing the gauge structure in doped antiferromagnetic materials has been dis-

cussed in ref. [34]. The presence of gauge interactions leads to induced parity-violating magnetic moments in effective low-energy theory models of high-temperature superconductors, under the influence of strong external magnetic fields, after integrating out fermionic degrees of freedom pertaining to higher Landau levels. These magnetic moments are induced even in the case where the nodal gap is *parity conserving*. As shown in [34], the scaling of the magnetic moment in the effective theory for the lowest Landau level, induced by the massive $SU(2)$ gauge bosons of the model of [7] in the gapped phase, is different from the scaling in the case where the moment is induced by the $U_S(1)$ interactions, as well as from the corresponding case of parity-violating Chern-Simons theories [35]. These are experimentally testable predictions, which provide independent probes of gauge interactions, that are complementary to the thermal conductivity method described here and in refs. [9, 23].

We are well aware that our effective theory analysis may be too crude to allow for a quantitative comparison with the realistic situations, but we believe that the proposed phase structure, and the associated non-trivial symmetry breaking mechanisms that characterise the model of [7], may capture essential features of the phase structure of high-temperature superconductivity, if the above scenaria are realised in nature.

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References

[1] D.A. Bonn *et al.*; *ibid.* **68** (1992), 2390;
C.C. Tsuei *et al.*, *Phys. Rev. Lett.* **73** (1994), 593;
K.A. Moler *et al.*, *Phys. Rev. Lett.* **73** (1994), 2744;

J. R. Kirtley *et al.*, *Nature* **373** (1995), 225;

D. Wollman *et al.*, *Phys. Rev. Lett.* **74** (1995), 797.

[2] A. Loeser *et al.*, *Science* **273** (1996), 325;

H. Ding *et al.*, *Nature* **382** (1996), B. Batlogg *et al.*, *Physica* **235-240 C** (1994), 130;

C.C. Homes *et al.*, *Phys. Rev. Lett.* **71** (1993), 1645;

A.V. Puchkov *et al.*, *Phys. Rev. Lett.* **77** (1996), 3212.

[3] K. Krishana *et al.*, *Science* **277** (1997), 83.

[4] P.A. Marchetti, Z.-B. Su and Lu Yu, cond-mat/9709109 and cond-mat/9805191.

[5] L. Balents, M. Fisher and C. Nayak, cond-mat/9803086.

[6] N. Dorey and N. E. Mavromatos, *Phys. Lett.* **B250** (1990), 107; *Nucl. Phys.* **B386** (1992), 614;

for a comprehensive review of this approach see: N.E. Mavromatos, *Nucl. Phys. B* (Proc. Suppl.) **C33** (1993), 145.

[7] K. Farakos and N.E. Mavromatos, *Phys. Rev.* **B57** (1998), 3017.

[8] J. Kosterlitz and D. Thouless, *J. Phys.* **C6** (1973), 1181.

[9] K. Farakos and N.E. Mavromatos, cond-mat/9710288, *Int. J. Mod. Phys.* **B12** (1998), 809.

[10] V.P. Gusynin, V.A. Miransky and I.A. Shovkovy, *Phys. Rev. Lett.* **73** (1994), 3499; *Phys. Rev.* **D52** (1995), 4718; *Nucl. Phys.* **B462** (1996), 249.

[11] G. Semenoff and L.C.R. Wijerwardhana, *Phys. Rev. Lett.* **64** (1989), 2633.

[12] I. Affleck, Z. Zou, T. Hsu and P.W. Anderson, *Phys. Rev.* **B38** (1988) 745.

[13] J. Fröhlich and P. Marchetti, *Phys. Rev.* **B46** (1992), 6535;

J. Fröhlich, T. Kerler and and P. Marchetti, *Nucl. Phys.* **B374** (1992), 511.

[14] A. Kovner and B. Rosenstein, *Mod. Phys. Lett.* **A5** (1990), 2661; *Phys. Rev.* **B42** (1990), 4748;

A. Kovner, B. Rosenstein and D. Eliezer, *Nucl. Phys.* **B350** (1990), 325.

[15] A.M. Polyakov, *Phys. Lett.* **59B** (1975), 79.

[16] I. Affleck, J. Harvey and E. Witten, *Nucl. Phys.* **B206** (1982), 413.

[17] K. Farakos, N.E. Mavromatos, hep-lat/9707027, *Mod. Phys. Lett.* **A**, in press;

K. Farakos, N.E. Mavromatos D. Mc Neill, OUTP-98-44P.

[18] R. Shankar, *Phys. Rev. Lett.* **63** (1989), 203; *Nucl. Phys.* **B330** (1990), 433.

[19] B. Rosenstein, B.J. Warr and S.H. Park, *Phys. Rev. Lett* **62** (1989), 1433;

G. Gat, A. Kovner, B. Rosenstein and B.J. Warr, *Phys. Lett* **B240** (1990), 158.

[20] R. D. Pisarski, *Phys. Rev.* **D29** (1984), 2423;

T. W. Appelquist, M. Bowick, D. Karabali and L. C. R. Wijewardhana, *Phys. Rev.* **D33** (1986), 3704.

T. W. Appelquist, D. Nash and L. C. R. Wijewardhana, *Phys. Rev. Lett.* **60** (1988), 2575.

I. J. R. Aitchison, N.E. Mavromatos and D. McNeill, *Phys. Lett.* **B402** (1997), 154;

K-I. Kondo and T. Murakami, *Phys. Lett.* **B410** (1997), 257.

[21] E. Dagotto, A. Kocic and J.B. Kogut, *Phys. Rev. Lett.* **62** (1989), 1083; *Nucl. Phys.* **B334** (1990), 279.

[22] A.M. Polyakov, *Gauge Fields and Strings* (Harwood 1987);
S. Deser and A.N. Redlich, *Phys. Rev. Lett.* **61** (1989), 1541;
N.E. Mavromatos and M. Ruiz-Altaba, *Phys. Lett.* **A142** (1989), 419;
B. Rosenstein and A. Kovner, *Nucl.Phys.* **B346** (1990), 576.

[23] G. Semenoff, I.A. Shovkovy and R. Wijerwadhana, hep-ph/9803371.

[24] N. Dorey and N. E. Mavromatos, *Phys. Rev.* **B44** (1991), 5286.

[25] G.A. Diamandis, B.C. Georgalas and N.E. Mavromatos, *Mod. Phys. Lett.* **A13** (1998), 387.

[26] P.B. Wiegmann, *Phys. Rev. Lett.* **60** (1988), 821;
S. Sarkar, *J. Phys.* **A23** (1990) L409; *J. Phys.* **A24** (1991), 1137;
F.H.L. Essler, V.A. Korepin and K. Schoutens, *Phys. Rev. Lett.* **68** (1992), 2960 ;
A. Lerda and S. Sciuto, *Nucl. Phys.* **B410** (1993), 577.

[27] A.V. Shpagin, preprint hep-ph/9611412.

[28] K. Farakos, G. Koutsoumbas and N.E. Mavromatos, hep-lat/9802037, *Phys. Lett.* **B** in press.

[29] J. Schwinger, *Phys. Rev.* **82** (1951), 664.

[30] A. Chodos, K. Everding and D.A. Owen, *Phys. Rev.* **D42** (1990), 2881.

[31] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, New York 1980).

[32] D.S. Lee, C.N. Leung and Y.J. Ng, *Phys. Rev.* **D55** (1997), 6504;
V.P. Gusynin and I.A. Shovkovy, *Phys. Rev.* **D56** (1997), 5156;
The above references pertain to finite-temperature analysis of four-dimensional QED, but the relevant three-dimensional gap equations are derived by appropriate dimensional reduction [9].

[33] R. Kubo, *Some aspects of the statistical-mechanical theory of irreversible processes*, in "Lectures in Theoretical Physics", V.1, ed. by W. E. Brittin and L. G. Dunham, (Interscience Publishers, New York, 1959).

[34] N.E. Mavromatos and Arshad Momen, hep-th/9802119, OUTP-98-11P.

[35] I. Kogan. *JETP* Lett. **49** (1989), 225;

M. Dobroliubov, D. Eliezer, I. Kogan, G. Semenoff and R. Szabo, *Mod. Phys. Lett.* **A8** (1993), 2177;

D.K. Hong and J.Y. Kim, *Phys. Lett.* **B383** (1996), 327.